Int.J.Curr.Microbiol.App.Sci (2020) Special Issue-11: 836-843

International Journal of Current Microbiology and Applied Sciences
ISSN: 2319-7706 Special Issue-11 pp. 836-843
Journal homepage: http://www.ijcmas.com

Original Research Article

A Study of Variation in Forecasting using Theil's Statistic

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Keywords

Absolute Error, Relative Error, Mean Squared Error, Relative Error, Thiel's statistic

A B S T R A C T

Numerous institutions and organizations, nowadays, are interested in focusing on forecasting for prediction and formulating future planning. Even in agriculture, we plan various dressings taking forecasting views under consideration. Exploring such avenues in planning for horticulture too, cannot be ruled out. For the purpose, we generally use past long-term data to have an idea about the future trend. Application of forecasting methods, in particular, mainly depends upon the problem under study and the associated situation(s). Such studies have vital applications and importance in agriculture. The crop production is beset by constraints such as drought, flooding, salt stress and extreme temperatures, all of which are expected to worsen with climate change. Drastic changes in rainfall patterns coupled with rising temperatures generally introduce unfavorable growing conditions (due to drought, flooding, etc.) into cropping calendars thereby modifying growing seasons which could subsequently reduce crop productivity. These factors cannot be controlled as such. Effect of temperature on the growth characters is a major factor. Various studies have been done to study the impact of these factors on the productivity of crops. In the present study, an attempt has been made to study various aspects of forecasting evaluation and obtaining Thiel's statistic on the observations. This attempt depicts statistical applications on comparison of actual observation with hypothetical series of forecasts through which different properties, formation and evaluation of hypothetical forecast have been analyzed.

Introduction

Forecasting is concerned with what the future will look like, rather than what it should look like. Forecasting is based upon the assumption that future trend is almost similar to the past. For administrative and management purposes, future planning and policies are framed using forecasts as inputs. According to Armstrong (1992), forecasting is the prediction or estimation of an actual value in future. Statistical evaluation of the performance of forecast is quite indispensible for inferring the future trend. There are many ways of drawing forecasting observations. The construction of forecasting can be done in many ways and evaluation and interpretation of the same are very vital in studying the trend. Average of lagging or leading data can be used as forecast observation and can be compared with the actual observation to find any correlation or covariance or any other trend between the two. We may observe an unintentional or imprudent deviation from an observed or calculated value to the true or actual observation. This deviation is known as an error i.e. it represents the amount of deviation from a standard value which might have occurred due to uncontrollable factors.

Besides crop response, there are many other factors like sun shine, wind flow, rainfall, temperature, humidity etc. on which the production of agriculture is based. Obviously, the agricultural production is greatly affected by these factors. Though, these factors, as such cannot be controlled, but variations due to these factors can be enumerated by reducing from them from the total variations. Various studies have been conducted on finding the effect of these uncontrollable factors on crop responses. Temperature, which may be considered as one of the most extraneous factors, is having a direct impact on the growth of crops. To have an idea of future trend of the temperature, we need to study the past long term data of the temperature. In the present study, an attempt has been done to investigate various aspects of forecasting evaluation and obtaining Thiel's statistic on the observations.

Materials and Methods

The month-wise meteorological data of maximum and minimum temperature of 2018 has been taken as actual and compared with hypothetical forecast of the observations and meteorological data of month-wise maximum and minimum temperature of last eight years (2010-2017 as hypothetical forecast) have been used for this research and collected by Department of Agrometeorology, Agriculture College, Govind Ballabh Pant University of Agriculture and Technology, Pantnagar, U.S Nagar (Uttarakhand). Pantnagar is situated in the Tarai belt, at latitude of 29.2°N, 79°E longitude and at an altitude of 243.80 m above the mean sea level. The climate of Pantnagar comprises of sub-humid to sub-tropical with hot dry summers and cool winters. Generally, the monsoon sets in around third week of June and lasts upto September end. The mean annual rainfall is 1433.4 mm. May is the hottest month of the year and temperature generally rises up to 45.5±1.5°C. However, minimum temperature can be low as 1.5±1.0°C in the month of January. A few showers generally occur during the winters and occasionally during the summer. Maximum relative humidity remains in the range of 90-95 percent which is experienced during monsoon season and also during winter season.

Results and Discussions

There are many statistical techniques for computing standard statistical measures by analyzing different forms of errors as enumerated below. For the present study, we have taken the actual monthly observations of maximum temperature of the year 2018 (Y_{t_1}) and the average of maximum temperature for the last eight years (2010 to 2017) observations as forecast observations (F_{t_1}). Similarly, actual monthly observations of minimum temperature of the year 2018 (Y_{t_2}) and the average of minimum temperature for eight years (2010 to 2017) observations as hypothetical forecast observations (F_{t_2}).

Mathematically, the error is computed by subtracting the forecast observations from the actual observations as given below:

Error = actual observations - forecast observations

The error e_{t_1} denotes maximum temperature and e_{t_2} denotes minimum temperature are calculated as under:

$$\text{Error}_1;\ e_{t_1} = Y_{t_1} - F_{t_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{Error}_2;\ e_{t_2} = Y_{t_2} - F_{t_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

According to Cook (2006), a low value of the mean error may conceal forecasting inaccuracy due to the offsetting effect of large positive and negative forecast errors. Arithmetic average of all the deviations, known as mean error is computed as sum of the values of the items divided by the number of items. As a measure of location, it is used to infer the accuracy of a forecast and have been computed by using the following formulae:

Mean Error; $ME_1 = \frac{1}{n}\sum_{t_1=1}^{n} e_{t_1}$ ..(3)

Mean Error; $ME_2 = \frac{1}{n}\sum_{t_1=1}^{n} e_{t_2}$ ..(4)

Again, there are two types of errors; the absolute error and the relative error. Both of these errors are affected by the accuracy of measuring tools. The absolute error denotes the magnitude of the difference between the exact value and the approximation or the measured value 0. The absolute value is required because sometimes the measurement could be smaller, giving a negative number. The absolute error plays a vital role for making predictions from model. Average of all the absolute errors is known as mean absolute error (MAE). The MAE for both, maximum and minimum temperatures has been calculated by using the following relationships:

Mean Absolute Error; $MAE_1 = \frac{1}{n}\sum_{t_1=1}^{n} |e_{t_1}|$ ……………………………………….(5)

Mean Absolute Error; $MAE_2 = \frac{1}{n}\sum_{t_2=1}^{n} |e_{t_2}|$ ……………………………...(6)

Makridakis *et al*.. (1998) define the mean squared error (MSE) as a measure of accuracy obtained by squaring the individual error for each item in a data set and taking arithmetic average of all of the sum of squares. The MSE gives greater weight to large errors than to small errors because of squaring. It is a single value that provides information about the goodness of fit. Smaller the MSE value, the better the fit, as smaller values imply smaller magnitudes of error. Suppose all observations lie exactly on the regression line resulting residual errors of 0, and the MSE calculation would also be 0, which is the smallest possible MSE value.

The MSE is having the advantage of being easy to calculate and handle mathematically. However, since a MSE is an absolute value instead of in percentage, it does not facilitate comparison across different time series of various time intervals. It also gives greater weight to large errors than to smaller ones because the errors are squared (magnified) before being summed. Since the ability of a forecasting methods to detect large errors, is often regarded as one the most important criteria, the MSE method has been popular for years. However, Armstrong (2001) argues that the MSE should not be used for forecast comparisons because it is not independent of scale and it is unreliable when compared to other measures. The smaller the means squared error, the closer it is to the line of best fit.

We have obtained the Mean Squared Error by taking the averages of all the squared errors i.e. the difference between the actual observation and the forecast values as following .

Mean Squared Error; $MSE_1 = \frac{1}{n}\sum_{t_1=1}^{n} (e_{t_1})^2$ ……………………………………..…..(7)

Mean Squared Error; MSE_2 $\frac{1}{n}\sum_{t_2=1}^{n} (e_{t_2})^2$ ……………………..…………………….…..(8)

On the other hand, the relative or percent error is defined as the absolute error relative to the size of the measurement and is obtained by dividing the absolute error to the magnitude of the exact value. The percent error is the relative error expressed in terms of per 100.

Percent or Relative Error;

$$PE_1 = \left(\frac{Y_{t_1} - F_{t_1}}{Y_{t_1}}\right) * 100 \quad \text{………..…..(9)}$$

Percent or Relative Error;

$$PE_2 = \left(\frac{Y_{t_2} - F_{t_2}}{Y_{t_2}}\right) * 100 \quad \text{……..…....(10)}$$

Mean Percentage Error$_1$; MPE_1 =

$$\frac{1}{n}\sum_{t_1=1}^{n} * PE_1 \quad \text{………..……………...(11)}$$

Mean Percentage Error$_2$; MPE_2 =

$$\frac{1}{n}\sum_{t_2=1}^{n} * PE_2 \quad \text{………..………………...(12)}$$

Makridakis *et al.*. (1998) elaborate on the mean percentage error (MPE) as it is the mean or average of the sum of all of the percentage errors for a given data set taken without regard to sign so as to avoid the problem of positive and negative values cancelling one another. Because of this, it is used as measure of a measure of bias in forecasting. Also, the

mean absolute percentage error (MAPE) is the mean or the of the sum of all the percentage errors without regard to the sign. It is considered as measure of accuracy commonly used in quantitative methods of forecasting.

Mean Absolute Percentage Error$_1$; $MAPE_1$ =

$$\frac{1}{n}\sum_{t_1=1}^{n} |PE_1| \quad \text{……………...(13)}$$

Mean Absolute Percentage Error$_2$; $MAPE_2$ =

$$\frac{1}{n}\sum_{t_2=1}^{n} |PE_2| \quad \text{……...……………………...(14)}$$

While computing different types of errors using methods listed above, almost all the errors correspond equal weightage except MSE, since errors are being squared in calculating it. And, it would be appropriate to have a measure that would enable to provide a relative basis for comparison. Such a measure was developed by Theil (1966). According to Makridakis *et al.*. (1998), ‘this statistic allows a relative comparison of formal forecasting methods with naïve approaches and also squares the errors involved so that large errors are given much more weight than small errors. The positive characteristic that is given up in moving to Theil’s U-statistics as a measure of accuracy is that of intuitive interpretation.’ However, prior to the development of this statistic, Theil (1958) proposed two different U statistics which were later used in finance. The first (U_1) is a measure of forecast accuracy whereas the second (U_2) is a measure of forecast quality concerning the accuracy and quality of forecasting. Theil proposed two error measures, but at different times and under the same symbol. U1 is taken from Theil (1958), where he calls U a measure of forecast accuracy by considering the actual observations and the corresponding predictions. It remained open whether the absolute values be used or as observed and predicted changes be taken as these are. Theil (1966) proposed U2 as a measure of forecast quality taking pair of predicted and observed changes.

The U-statistic developed by Theil (1966) is an accuracy measure that emphasizes the importance of large errors (as in MSE) as well as providing a relative basis for comparison with naïve forecasting methods. Theil's U. Theil's U statistic is a relative accuracy measure that compares the forecasted results with the results of forecasting with minimal historical data. It also squares the deviations to give more weight to large errors and to exaggerate errors, which can help eliminate methods with large errors.

Makridakis *et al.,* (1998) have simplified Theil's equation to the form shown below:

Theil's U-statistic for maximum temperature:

$$U_1 = \sqrt{\frac{\sum_{t_1=1}^{n-1}\left(FPE_{t_1+1} - APE_{t_1+1}\right)^2}{\sum_{t_1=1}^{n-1}\left(APE_{t_1+1}\right)^2}}$$

……………………………………......…...(15)

Where $FP^{E_{t_1+1}} = \frac{F_{t_1+1} - Y_{t_1}}{Y_{t_1}}$ ………...(16)

And $AP^{E_{t_1+1}} = \frac{Y_{t_1+1} - Y_{t_1}}{Y_{t_1}}$ ……..……...(17)

where $FP^{E_{t_1+1}}$ and $AP^{E_{t_1+1}}$ donates the forecast and actual relative changes in the maximum temperatures.

And Theil's U-statistic for minimum temperature is given by:

$$U_2 = \sqrt{\frac{\sum_{t_2=1}^{n-1}\left(FPE_{t_2+1} - APE_{t_2+1}\right)^2}{\sum_{t_1=2}^{n-1}\left(APE_{t_2+1}\right)^2}}$$

………..(18)

Where $FP^{E_{t_2+1}} = \frac{F_{t_2+1} - Y_{t_2}}{Y_{t_2}}$ …....…..(19)

and $AP^{E_{t_2+1}} = \frac{Y_{t_2+1} - Y_{t_2}}{Y_{t_2}}$ …………....(20)

where $FP^{E_{t_2+1}}$ and $AP^{E_{t_2+1}}$ donate the forecast and actual relative changes in the minimum temperatures.

Theil's U-statistic can be interpreted as dividing the RMSE (Root Mean Square Error, or square root of the MSE) of the proposed forecasting method by the RMSE of a no-change (naïve, U=1) model. If U is equal to 1, it means that the proposed model is as good as the naïve model. If U is greater than 1, there is no point in using the proposed forecasting model since a naïve method would produce better results. It is worthwhile to consider using the proposed model only when U is smaller than 1 (the smaller the better), indicating that more accurate forecasts than a no-change model can be obtained.

In conclusion, as we can observe that the means of maximum temperature of actual and forecast observations are 29.67 and 29.68 respectively resulting a marginal change (0.01). Similarly, the mean values of actual and forecast observations are 16.73 and 17.30 with a difference of 0.57 can be seen for the same period in minimum temperature. In both the cases, we see a low value of mean error which indicates forecasting inaccuracy due to large positive and negative forecast errors. Absolute values are used to overcome this inaccuracy which may arise due to large positive and negative forecast errors. It should be noted that the mean error provides useful information on the bias of the actual forecast errors. The average squared error and the average absolute error may surpass the cancellation of positive and negative errors limitation of the mean error, but eventually, MSE and MAE fail to provide information on forecasting accuracy relative to the scale of the series examined. The average absolute error (1.03) and average squared error (1.81) can be noted in the actual and forecast observations for maximum temperature while for the minimum temperature; the average absolute error and average squared error were found to be 080 and 1.27 respectively. Again, the maximum temperature average percent error was found to be 4.20 and this was almost double for minimum temperature average percent error compared for the same period.

Table.1 Measurement of Errors in Actual & Forecast Values of Maximum Temperature (°C)

Period		Observation	Forecast	Error	Absolute Error	Squared Error	Percent Error	Absolute Percent Error
		Y_{t_1}	F_{t_1}	e_{t_1}	AE_1	SE_1	PE_1	APE_1
Jan.	1	16.70	18.69	-1.99	1.99	3.96	-11.92	11.92
Feb.	2	24.60	21.64	2.96	2.96	8.76	12.03	12.03
Mar.	3	30.90	29.21	1.69	1.69	2.86	5.47	5.47
Apr.	4	34.50	35.45	-0.95	0.95	0.90	-2.75	2.75
May.	5	36.80	36.85	-0.05	0.05	0.00	-0.14	0.14
Jun.	6	35.60	36.45	-0.85	0.09	0.72	-2.39	2.39
Jul.	7	33.10	32.33	0.77	0.77	0.59	2.33	2.33
Aug.	8	30.80	32.25	-1.45	1.45	2.10	-4.71	4.71
Sep.	9	31.50	32.31	-0.81	0.81	0.66	-2.57	2.57
Oct.	10	30.90	31.39	-0.49	0.49	0.24	-1.59	1.59
Nov.	11	27.50	27.40	0.10	0.10	0.01	0.36	0.36
Dec.	12	23.10	22.13	0.97	0.97	0.94	4.20	4.20
Means		29.67	29.68	-0.01	1.03	1.81	-0.14	4.20

Table.2 Measurement of Errors in Actual & Forecast Values of Minimum Temperature (°C)

Period		Observation	Forecast	Error	Absolute Error	Squared Error	Percent Error	Absolute Percent Error
		Y_{t_2}	F_{t_2}	e_{t_2}	AE_2	SE_2	PE_2	APE_2
Jan.	1	5.60	7.19	-1.59	1.59	2.53	-28.39	28.39
Feb.	2	8.90	9.19	-0.29	0.29	0.08	-3.26	3.26
Mar.	3	11.80	13.09	-1.29	1.29	1.66	-10.93	10.93
Apr.	4	17.70	17.98	-0.28	0.28	0.08	-1.58	1.58
May.	5	22.30	21.95	0.35	0.35	0.12	1.57	1.57
Jun.	6	25.90	25.46	0.44	0.44	0.19	1.70	1.70
Jul.	7	26.00	25.76	0.24	0.24	0.06	0.92	0.92
Aug.	8	25.30	25.59	-0.29	0.29	0.08	-1.15	1.15
Sep.	9	23.90	23.89	0.01	0.01	0.00	0.04	0.04
Oct.	10	15.50	18.13	-2.63	2.63	6.92	-16.97	16.97
Nov.	11	11.70	11.40	0.30	0.30	0.09	2.56	2.56
Dec.	12	6.10	7.94	-1.84	1.84	3.39	-30.16	30.16
Means		16.73	17.30	-0.57	0.80	1.27	-7.14	8.27

Table.3 Theil's U-statistics for Maximum Temperature (°C)

Period		Observation		Forecast				U-Statistic
		Y_{t_1}	$Y_{t_{1+1}}$	F_{t_1}	$F_{t_{1+1}}$	$FPE_{t_{1+1}}$	$APE_{t_{1+1}}$	
Jan.	**1**	16.7	24.6	18.69	21.64	0.30	0.47	0.37
Feb.	**2**	24.6	30.9	21.64	29.21	0.19	0.26	0.27
Mar.	**3**	30.9	34.5	29.21	35.45	0.15	0.12	0.26
Apr.	**4**	34.5	36.8	35.45	36.85	0.07	0.07	0.02
May.	**5**	36.8	35.6	36.85	36.45	-0.01	-0.03	0.71
Jun.	**6**	35.6	33.1	36.45	32.33	-0.09	-0.07	0.31
Jul.	**7**	33.1	30.8	32.33	32.25	-0.03	-0.07	0.63
Aug.	**8**	30.8	31.5	32.25	32.31	0.05	0.02	1.16
Sep.	**9**	31.5	30.9	32.31	31.39	0.00	-0.02	0.82
Oct.	**10**	30.9	27.5	31.39	27.4	-0.11	-0.11	0.03
Nov.	**11**	27.5	23.1	27.4	22.13	-0.20	-0.16	0.22
Dec.	**12**	23.1	23.5	22.13	18.71	--	--	--
Means		29.67	30.23	29.68	29.68	0.03	0.30	0.44

Table.4 Theil's U-statistics for Minimum Temperature (°C)

Period		Observation		Forecast				U-Statistic
		Y_2	$Y_{t_{2+1}}$	F_{t_2}	$F_{t_{2+1}}$	$FPE_{t_{2+1}}$	$APE_{t_{2+1}}$	
Jan.	**1**	5.6	8.9	7.19	9.19	0.64	0.59	0.09
Feb.	**2**	8.9	11.8	9.19	13.09	0.47	0.33	0.44
Mar.	**3**	11.8	17.7	13.09	17.98	0.52	0.50	0.05
Apr.	**4**	17.7	22.3	17.98	21.95	0.24	0.26	0.08
May.	**5**	22.3	25.9	21.95	25.46	0.14	0.16	0.12
Jun.	**6**	25.9	26	25.46	25.76	-0.01	0.00	2.40
Jul.	**7**	26	25.3	25.76	25.59	-0.02	-0.03	0.41
Aug.	**8**	25.3	23.9	25.59	23.89	-0.06	-0.06	0.01
Sep.	**9**	23.9	15.5	23.89	18.13	-0.24	-0.35	0.31
Oct.	**10**	15.5	11.7	18.13	11.4	-0.26	-0.25	0.08
Nov.	**11**	11.7	6.1	11.4	7.94	-0.32	-0.48	0.33
Dec.	**12**	6.1	2.9	7.94	7	--	--	--
Means		16.73	16.50	17.30	17.28	0.11	0.58	0.39

The Thiel's U-statistic computed for maximum temperatures of actual and forecast observations is ranging from 0.02 to 1.16 with a mean value of 0.44. Similarly, Thiel's U-statistic computed for minimum temperatures of actual and forecast observations is ranging from 0.01 to 2.4 with a mean value of 0.39. In fact, the more accurate the forecasts, the lower the value of the U statistic. The U statistic, preferably be in between 0 and 1, with values closer to 0 indicating greater forecasting accuracy. If it is more than 1, we conclude that forecasting is not so perfect and this imperfection increases as it reaches 1. On the basis of the above computations, it can be noticed that U-statistic computed for minimum temperature of actual and forecast observations is better and represents the 'best' set of forecasts as compared to the U-statistic computed for maximum temperature of actual and forecast observations for the same period.

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